Eddy Viscosity in Quiescent and Coflowing Axisymmetric Jets

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Introduction

THE objective of this investigation was to show how one of the more successful eddy viscosity models[1–3] for coflowing axisymmetric jets, presented in Eq. (1), may be modified such that a more general description of mixing in both quiescent and coflowing jets is obtained

$$\epsilon = \frac{0.036}{a} \int_0^\infty |\bar{\rho}\bar{U} - \rho_e U_e| \, rdr \tag{1}$$

where, $\bar{\rho}$ = the mean density, $\bar{U}$ = the mean axial velocity, $(\ )_e$ = freestream, r = the radial coordinate, a = the initial radius of the jet.

The most important point in favor of applying eddy viscosity models to problems in turbulent mixing is that at this time it is the only model available for analysis of problems of engineering interest, e.g., a high-speed hydrogen jet exhausting into a hot air stream. Its most significant shortcoming is that it requires the shear stress be described in terms of local flow parameters, i.e., neglect upstream history effects. There have been attempts to incorporate history effects into the model by introducing an additional equation for either the Reynolds stress or eddy viscosity.[4,5] But these approaches are of little use until a means of specifying the initial shear stress distributions is obtained, e.g., a model such as eddy viscosity.

Equation (1) was developed from an extension of Clauser's model for the wake region of a turbulent boundary layer to axisymmetric free shear layers. When the mass defect across a shear layer is small, but the viscosity gradient is not, the model erroneously predicts a vanishing eddy viscosity. However, this limitation must be accepted if the mass defect concept is to be retained. The subject of this Note is a generalization of the eddy viscosity given by Eq. (1) to make it apply to both quiescent and coflowing axisymmetric shear layer flows.

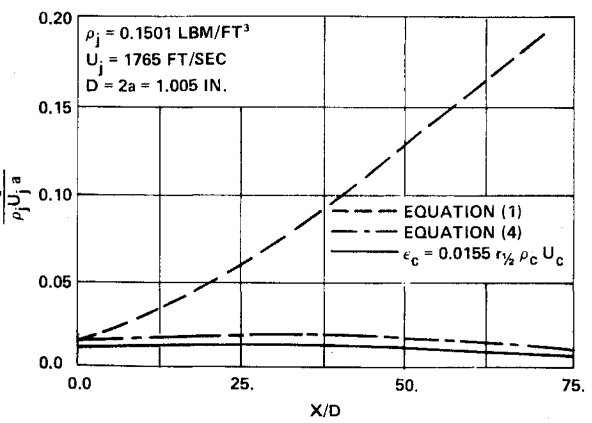

Fig. 1 Eddy viscosity models for Mach 2.2 quiescent jet, Ref. 6.

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Quiescent Jet

Eggers[6] has measured the velocity field for a Mach 2.2 quiescent air jet from exit to 75 diam downstream. Also reported were the eddy viscosity values that would most accurately predict the downstream velocity field using the method of Warren.[7] The eddy viscosities determined by Eggers and those obtained from Eq. (1) are shown in Fig. 1. It will be shown that the eddy viscosity predicted by Eq. (1) greatly overestimates the value required to best describe the downstream velocity field.

Generalized Model

It is well known that turbulent jets become similar and self-preserving far downstream,[8] i.e., velocity, shear and consequently eddy viscosity are completely described by the local conditions. Assuming incompressibility (i.e., $\bar{\rho} = \rho_e$)

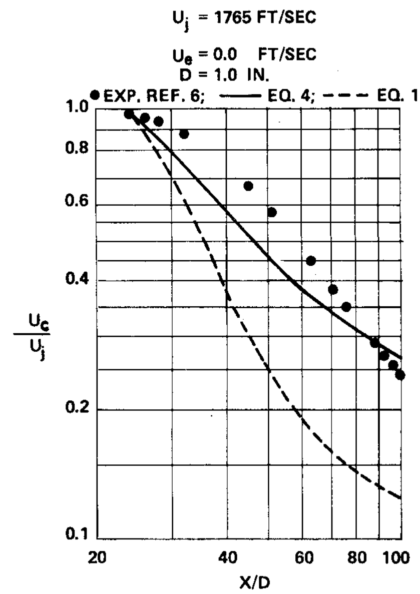

Fig. 2a Centerline velocity decay Ref. 6, calculations started at X/D = 23.0.

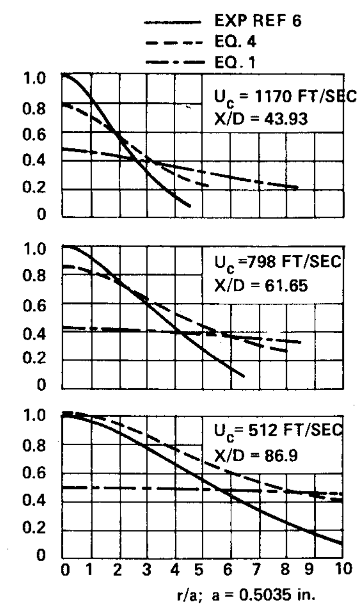

Fig. 2b Radial distribution of axial velocity, Ref. 6, calculations started at X/D = 22.92.

and taking

$$\bar{U} - U_e = (U_c - U_e)f(\eta) \tag{2}$$

where $(\ )_c$ denotes centerline value, $r_{1/2}$ = the half radius, $\eta = r/r_{1/2}$, and $f(\eta)$ = a similarity function. Substituting into Eq. (1) yields

$$\epsilon = \left(\frac{0.036}{a}\right) r^2_{1/2}\rho \, |U_c - U_e| \int_0^\infty f(\eta)\eta d\eta \tag{3}$$

It is seen that the characteristic length, a, in Eq. (1) is more appropriately identified with the half width of the flow rather than the initial jet radius if the condition of self-preservation is to be eventually satisfied at long distances downstream. In addition, the model will reduce to the Prandtl type form which has been shown to yield accurate predictions of the velocity field for quiescent jets[6,9] in the similarity region. Therefore, in order to improve the model, the characteristic length, a, was replaced by $r_{1/2}$ and the following relationship for eddy viscosity is proposed.

$$\epsilon = \frac{0.036}{r_{1/2}} \int_0^\infty |\bar{\rho}\bar{U} - \rho_e U_e| \, r dr \tag{4}$$

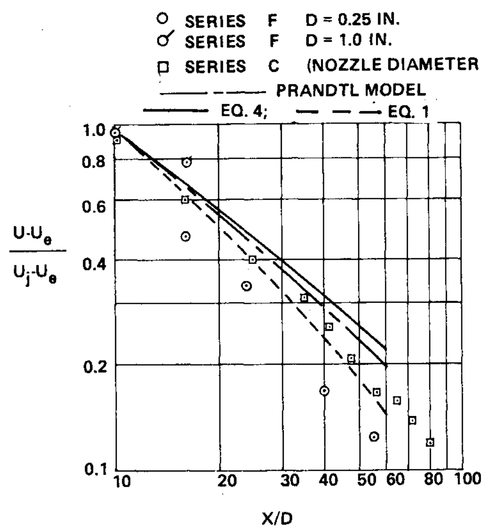

Fig. 3a Centerline velocity decay, Ref. 10, series F and C, calculations started at X/D = 10.0.

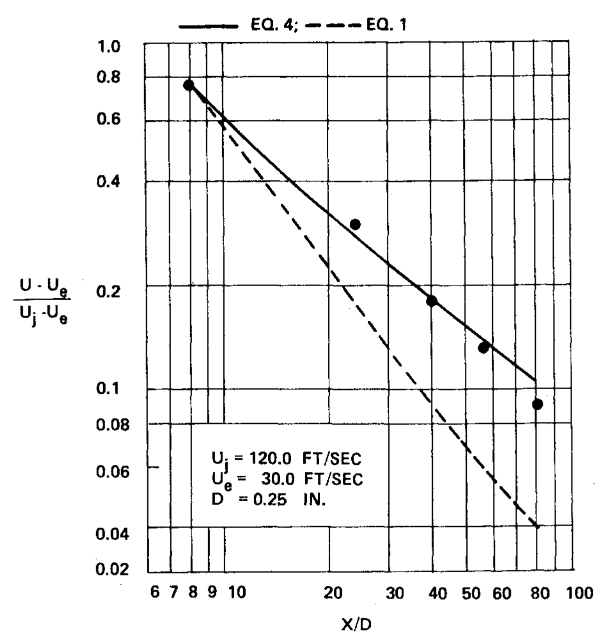

Fig. 3b Centerline velocity decay, Ref. 10, series E calculations started at X/D = 8.0.

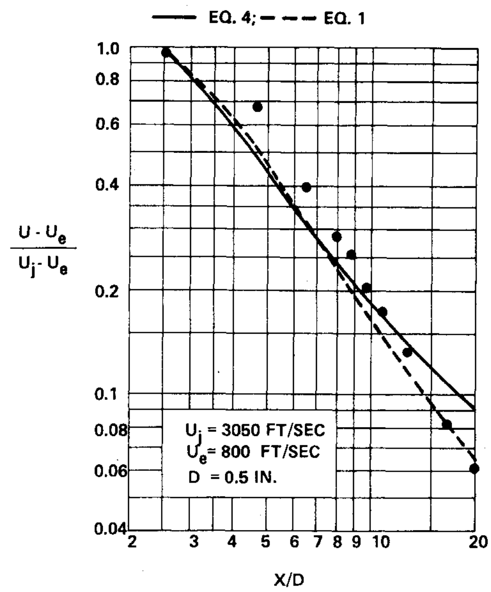

Fig. 4 Centerline velocity decay, Ref. 11, calculations started at X/D = 2.55.

which differs from Eq. (1) only in the choice of the characteristic length.

The eddy viscosity predicted by Eq. (4) is shown in Fig. 1. Clearly this eddy viscosity gives better agreement than Eq. (1) with the values found by Eggers[6] to accurately predict the downstream mixing region; the reason being $r_{1/2}$ increases with x. This fact is substantiated in Figs. 2a and 2b where the predictions of the velocity are compared with the quiescent jet data of Ref. 6. These predictions were obtained from a finite-difference solution of the boundary-layer equations. The equations were solved in von Mises coordinates; a finite but small freestream velocity (5% of the maximum velocity) was required to compute the quiescent jet cases. The complexity of the region immediately downstream of injection (the core region) requires special consideration in any jet mixing analyses and was not treated in this investigation, i.e., calculations were started downstream of the core.

Although these results show Eq. (4) is more successful in predicting quiescent jets than Eq. (1), it is necessary to apply this model to coflowing jets to evaluate its generality. Eq. (1) was shown by Schetz[1] to give good correlation with the data of Forstall[10] for the cases where the ratio of primary jet velocity to the freestream velocity, U_j/U_e = 2.0. In addition, Ref. 1 has also shown that the Prandtl model underestimates mixing in this particular case. Figure 3a

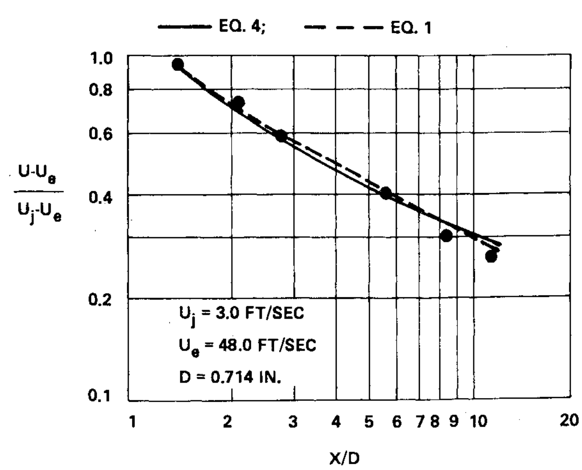

Fig. 5 Centerline velocity decay, Ref. 12, calculation started at X/D = 1.4.

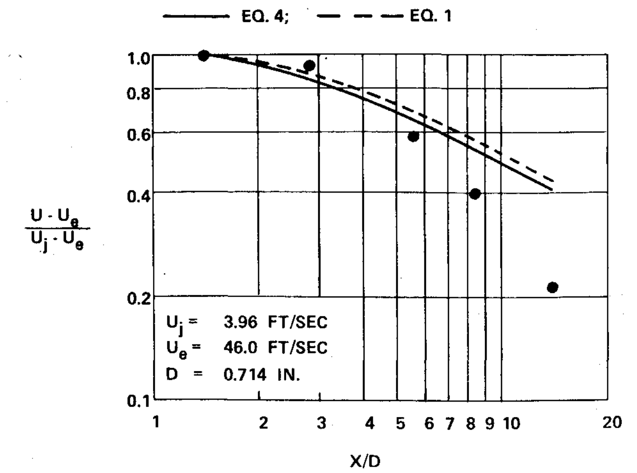

Fig. 6 Centerline velocity decay, Ref. 12, calculations started at X/D = 1.4.

shows somewhat better correlation of the data obtained with Eq. (1) than either the Prandtl model or Eq. (4).

On the other hand the usefulness of these comparisons made in Ref. 1 and shown in Fig. 3a is questionable for two important reasons. First, all the data shown was not taken with the same nozzle diameter (both D = 1.0 in. and 0.25 in. were used), and also the primary and secondary velocities were not fixed, i.e., in series C, $U_j = 180$ fps; whereas, in series F, $U_j = 90$ fps. Therefore, using the data from two different experimental cases to compare against the predictions of single case assumes the flowfield to be similar. Second, Forstall[10] reported the initial boundary layers on the splitter plate significantly increased the mixing. This effect is not considered in either of these models. An assumption of similarity is not required if only data points obtained with a 0.25-in.-diam nozzle are used in Series E of the Forstall data. When this is done, as shown in Fig. 3b, Eq. (1) overestimates mixing by as much as 50%. This overestimation is not surprising since this case more closely approximates the quiescent jet condition, i.e., $U_j = 4U_e$.

Examination of Fig. 4 shows that Eq. (4) also gave good agreement with the data of Chriss,[11] i.e., a high speed subsonic hydrogen jet exhausting into a high speed subsonic air stream. Both models give reasonable agreement with the low speed wake like ($U_e > U_j$) data of Ref. 12 for constant density flows (Fig. 5) but neither model did well (Fig. 6) for flows where the central jet gas was Freon and the external stream was air. Further verification of the generality of Eq. (4) was shown by comparison with 17 cases as reported in Ref. 14.

Discussion

Inspection of Figs. 2–5 shows that in some cases Eqs. (1) and (4) have predicted a centerline velocity decay which differs from experiment by as much as 50%. Two possible sources of this poor correlation are: first, the implicit assumption of equilibrium, and second, the absence of radial variation in the models. Radial variation was found to be significant[15] for flows in the transition region such as the data of Chriss, in which large density variations exist across the jet.

It is felt that better prediction methods for the entire mixing region must include the effects of upstream history of the turbulence quantities, i.e., intensities and shear correlations. Since the upstream flowfield development is important, these prediction schemes should begin at least at the point at which mixing is initiated. This region contains information about splitter plate boundary layer profiles and preturbulence levels which is neither measured nor reported in available jet mixing data. It is strongly recommended that future investigations include these important parameters.

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Plane Stress Analysis of Two Rigid Circular Inclusions

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Introduction

The problems of finding the stress field around an arbitrary hole in an infinite plane can be generally solved by mapping the given region into a unit circle via Schwartz-Christoffel transformation, and many recent studies have been reviewed extensively in the book by Savin.[1] However, this procedure becomes rather cumbersome for a hole with nonsmooth contour or one that is defined in a multiply connected region since the mapping function, in general, becomes an infinite series. To avoid an infinite number of terms,

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